An examination of the factored form of the derivative shows that $\gamma = 120°$ gives a minimum. Incidentally,

$$t = 4\sqrt{\frac{p}{3g\sqrt{3}}}, \qquad s = \tfrac{4}{3}p.$$

Also solved by T. M. BLAKSLEE, R. A. JOHNSON, H. L. OLSON, ARTHUR PELLETIER, S. W. REAVES, J. B. REYNOLDS, and ELIJAH SWIFT.

2794 [1919 458]. Proposed by B. J. BROWN, Kansas City, Mo.

Find the value of $x^{e^x} \div x^{x^x}$ when $x \doteq 0$ and when $x \doteq \infty$. I. C. S. 1902.

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

A^{b^c} is taken to mean $A^{(b^c)}$.

(i) When $x \doteq 0$, it is well known that $e^x \doteq 1$, $x^x \doteq 1$;

$$x (\log x)^n \doteq \frac{(\log x)^n}{1/x} \doteq -n\frac{(\log x)^{n-1}}{1/x} \doteq \cdots \doteq \pm n!\, x \doteq 0. \qquad y = x^{e^x} \div x^{x^x} = x^{e^x - x^x}.$$

$$\log y = (e^x - x^x) \log x = \frac{e^x - x^x}{1/\log x} \doteq \frac{0}{0} \doteq \frac{e^x - x^x(1 + \log x)}{-\dfrac{1}{x(\log x)^2}}$$

$$\doteq -xe^x(\log x)^2 + x^x(x(\log x)^2 + x(\log x)^3 \doteq 1.0 + 1(0 + 0) = 0.$$

Hence, as $x \doteq 0$, $y \doteq 1$.

(ii) When $x \doteq \infty$,

$$x^x \div e^x = \left(\frac{x}{e}\right)^x \doteq \infty;$$

hence,

$$e^x - x^x = e^x\left(1 - \left(\frac{x}{e}\right)^x\right) \doteq -\infty;$$

i.e., with the notation of (i), $\log y = (e^x - x^x) \log x \doteq (-\infty)(+\infty) \doteq -\infty$, or $y \doteq 0$.

2795 [1919, 458]. Proposed by C. N. SCHMALL, New York City.

A square is described touching the ellipse, $x^2/a^2 + y^2/b^2 = 1$, at the ends of its minor axis; a second ellipse is drawn circumscribing the square and tangent to the given ellipse at the ends of the major axis. The new ellipse is treated as the first and the process is continued until there are n new ellipses. Show that the last ellipse is a circle if the eccentricity of the original ellipse is $\sqrt{n/(n+1)}$.

SOLUTION BY GERTRUDE I. MCCAIN, Oxford, Ohio.

If x', y' be the coördinates of a corner of the first square, then each equals b; and, lying on the second ellipse,

$$\frac{x'^2}{a^2} + \frac{y'^2}{b_1^2} = 1,$$

where b_1 is the minor axis of the first circumscribed ellipse. Substituting b for x' and y' and solving for b_1^2,

$$b_1^2 = \frac{a^2b^2}{a^2 - b^2}.$$

Similarly,

$$b_2^2 = \frac{a^2b_1^2}{a^2 - b_1^2} = \frac{a^2b^2}{a^2 - 2b^2},$$

and

$$b_n^2 = \frac{a^2b^2}{a^2 - nb^2}.$$